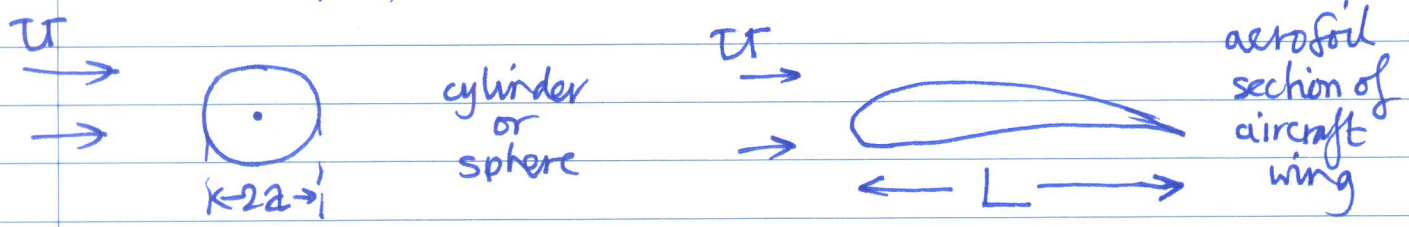
Chapter Three REYNOLDS NUMBER, THE VORTICITY EQUATION AND THE ROLE OF IRRATIONAL FLOW.

§3.1 Reynolds Number R

- It was noted in §2.1 that the governing equations in (2.1) are non-linear in $\underset{\sim}{u}$. Except in special cases (e.g. Chapter Two & Examples Sheet 2), this makes it impossible to obtain solutions of the NS equations in algebraic form.
  Thus (for example), there is no (known) algebraic solution of (2.1) for flow round a cylinder or a sphere that also satisfies the no-slip condition (1.20).



- Progress can be made by considering approximations based on physics.
- Begin by considering orders of magnitude of terms in NS equations.

$$\frac{\partial \underset{\sim}{u}}{\partial t} + (\underset{\sim}{u}\cdot\nabla)\underset{\sim}{u} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\underset{\sim}{u}.$$

$\frac{\partial \underset{\sim}{u}}{\partial t}$: important in unsteady flows.

$(\underset{\sim}{u}\cdot\nabla)\underset{\sim}{u}$: of order U^2/L

$-\frac{1}{\rho}\nabla P$: (nearly) always important.

$\nu\nabla^2\underset{\sim}{u}$: of order $\nu U/L^2$

- These simple estimates suggest that

$$\frac{\text{size of } (\underset{\sim}{u}\cdot\nabla)\underset{\sim}{u}}{\text{size of } \nu\nabla^2\underset{\sim}{u}} \sim \frac{U^2/L}{\nu U/L^2} = \frac{UL}{\nu}. \qquad (3.1)$$

($\sim$ = is of order)

- The quantity UL/ν where U is a typical speed, L is a typical length and ν is the kinematic viscosity is of great importance in fluid mechanics and is known as the Reynolds Number R (Reynolds $\approx$ 1890)

$$R = \frac{UL}{\nu} \qquad (3.2)$$

R is a pure number, independent of the units used. (R is dimensionless)