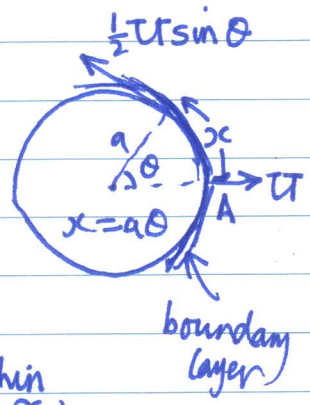


# Chapter Four BOUNDARY LAYERS

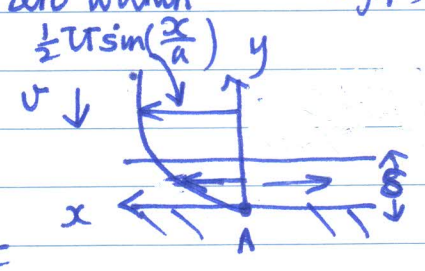
## § 4.1 Basic ideas

- Need to reconcile
  - (i) for  $R \gg 1$ ,  $|\nabla^2 \underline{u}|$  is apparently much less than  $|(\underline{u} \cdot \nabla) \underline{u}|$ , with
  - (ii) however large  $R$  is, the no-slip condition (1.20) is always satisfied.
- The reconciliation is due to Prandtl (1904). Read Acheson, pp 260-5, Batchelor, pp 302-8.
- For real flows with  $R \gg 1$ , exact solutions (eg Example 14; Example Sheet 3, Question 3 (+Question 7?) suggest that the effects of viscosity are confined to a thin layer - a boundary layer - under certain circumstances. The key points are:
  - (a) within the boundary layer there is diffusion of vorticity,
  - but (b) the normal component of velocity inhibits the growth of the layer of non-zero vorticity.

• In Example 15, we found that the irrotational flow round a sphere of radius  $a$  translating with speed  $U$  gave a slip velocity  $\frac{1}{2} U \sin \theta = \frac{1}{2} U \sin(\frac{x}{a}) = U(x)$ .  
 Then  $\frac{dU}{dx} = \frac{U}{2a} \cos(\frac{x}{a}) > 0$  for  $0 < x < \frac{\pi}{2} a$



The slip velocity cannot be tolerated and falls to zero within a thin boundary layer (see sketch). Since  $\frac{dU}{dx} > 0$  for  $0 < x < \frac{\pi}{2} a$ , so also must  $\frac{\partial u}{\partial x}$  be positive (at least at the edges of the boundary layer). Hence  $\frac{\partial v}{\partial y} < 0$ , and  $v < 0$  (at least at the edges of the boundary layer) since  $v = 0$  at  $y = 0$ . Since  $v$  is negative, convection of vorticity is towards  $y = 0$ , as required.



- We shall quantify these ideas for steady 2D flow and derive the appropriate form of the boundary-layer equations. The results can easily be generalised.

## § 4.2 The steady 2D boundary-layer equations

- We postulate the existence of an external stream  $\underline{u} = (U(x), 0, 0)$

