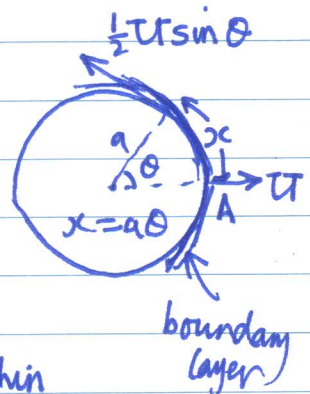


Chapter Four BOUNDARY LAYERS

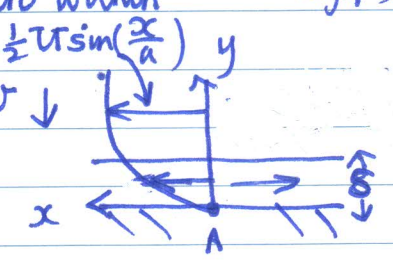
§ 4.1 Basic ideas

- Need to reconcile
 - (i) for $R \gg 1$, $|\nabla^2 \underline{u}|$ is apparently much less than $|(\underline{u} \cdot \nabla) \underline{u}|$, with
 - (ii) however large R is, the no-slip condition (1.20) is always satisfied.
- The reconciliation is due to Prandtl (1904). Read Acheson, pp 260-5, Batchelor, pp 302-8.
- For real flows with $R \gg 1$, exact solutions (eg Example 14; Example Sheet 3, Question 3 (+Question 7?) suggest that the effects of viscosity are confined to a thin layer - a boundary layer - under certain circumstances. The key points are:
 - (a) within the boundary layer there is diffusion of vorticity,
 - but (b) the normal component of velocity inhibits the growth of the layer of non-zero vorticity.

• In Example 15, we found that the irrotational flow round a sphere of radius a translating with speed U gave a slip velocity $\frac{1}{2} U \sin \theta = \frac{1}{2} U \sin(\frac{x}{a}) = U(x)$.
 Then $\frac{dU}{dx} = \frac{U}{2a} \cos(\frac{x}{a}) > 0$ for $0 < x < \frac{\pi}{2} a$



The slip velocity cannot be tolerated and falls to zero within a thin boundary layer (see sketch). Since $\frac{dU}{dx} > 0$ for $0 < x < \frac{\pi}{2} a$, so also must $\frac{\partial u}{\partial x}$ be positive (at least at the edges of the boundary layer). Hence $\partial v / \partial y < 0$, and $v < 0$ (at least at the edges of the boundary layer) since $v = 0$ at $y = 0$. Since v is negative, convection of vorticity is towards $y = 0$, as required.



- We shall quantify these ideas for steady 2D flow and derive the appropriate form of the boundary-layer equations. The results can easily be generalised.

§ 4.2 The steady 2D boundary-layer equations

- We postulate the existence of an external stream $\underline{u} = (U(x), 0, 0)$

