1. Let $\mathbf{a} = (3, 4, 5)$ and $\mathbf{b} = (-4, 3, 0)$ be vectors. What is the ratio of their lengths, $|\mathbf{a}|/|\mathbf{b}|$? Evaluate the scalar ('dot') product $\mathbf{a} \cdot \mathbf{b}$ and the vector ('cross') product $\mathbf{a} \times \mathbf{b}$. What can you conclude about $\mathbf{a}$ and $\mathbf{b}$?

2. Find $\nabla\phi$, when ϕ is each of the following:

   (i) $xy^2z^3 - x^3y^2z + 3a^6$, where a is a constant;

   (ii) $\ln r$, where $r = (x^2 + y^2 + z^2)^{1/2}$;

   A surface is given by $\phi = c$ where c is a constant. For case (i), what is the value of c for a surface that passes through the point $(-a, a, a)$. Find a unit normal vector to the surface at this point.

3. Let P be a point with $\vec{OP} = (x, y, z)$ that lies on the surface defined by $\phi = \phi(x, y, z) = c$, where c is a constant. Describe as precisely as possible the surface (sketch or in words) for the following ϕ:
   (*By default, assume that we work in three dimensions in this course!*)

   (i) $x^2 + y^2 + z^2$;   (ii) $(x^2 + y^2 + z^2)^{-3/2}$;   (iii) $x^2 + y^2$;
   (iv) $y^2 + z^2$;   (v) $y^2 - 4ax$,
   where a is a positive constant.

4. *Motion of a pendulum.* Consider a pendulum of length l, with a bob of mass m released from rest at an angle θ with the vertical (where θ is small). Sketch the forces acting on the bob. Using that $\sin\alpha \approx \alpha$ for small α, show that for a small horizontal displacement x the lateral restoring force is approximately $mg\,\theta \approx mg\,x/l$. Using Newton's second law, find a second-order ordinary differential equation for the displacement, and show that (in the absence of air resistance), the bob undergoes simple harmonic motion with frequency $\omega = \sqrt{g/l}$.

5. *Circular motion.* Recall that for circular motion, $a = \omega^2 r$. Consider a bucket of water being whirled in a vertical circle with a radius of 1m. Estimate the minimum angular frequency required to keep the water in the bucket. If the bucket is released at the top of the circle 2m above the ground, how far will it travel horizontally before hitting the ground?