MAS280

Example Sheet 1.

1. $\underline{a} = (3,4,5)$, $\underline{b} = (-4,3,0)$

$$\Rightarrow |\underline{a}| = \sqrt{3^2+4^2+5^2} = \sqrt{50}$$
$$|\underline{b}| = \sqrt{(-4)^2+3^2+0^2} = \sqrt{25}$$

$|\underline{a}|/|\underline{b}| = 5\sqrt{2}/5 = \sqrt{2}$.

$\underline{a}\cdot\underline{b} = 3\times(-4) + 4\times 3 + 5\times 0 = 0 \Rightarrow \perp r$

(ORTHOGONAL OR PERPENDICULAR)

$$\underline{a}\wedge\underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 4 & 5 \\ -4 & 3 & 0 \end{vmatrix} = (4\times 0 - 5\times 3)\underline{i} - (3\times 0 - 5\times(-4))\underline{j} + (3\times 3 - 4\times(-4))\underline{k}$$
$$= -15\underline{i} - 20\underline{j} + 25\underline{k} = (-15, -20, +25).$$

2. Recall $\nabla\varphi = \left(\frac{\partial}{\partial x}\varphi, \frac{\partial}{\partial y}\varphi, \frac{\partial}{\partial z}\varphi\right)$

(i) $\nabla\varphi = (y^2z^3 - 3x^2y^2z)\underline{i} + (2xyz^3 - 2x^3yz)\underline{j} + (3xy^2z^2 - x^3y^2)\underline{k}$

ii) See W, Ex. 1.2(B) : $\frac{\partial r}{\partial x} = \frac{x}{r}$

By chain rule $\frac{\partial\varphi}{\partial x} = \frac{\partial\varphi}{\partial r}\frac{\partial r}{\partial x} = \frac{d}{dr}(\ln r)\cdot\frac{x}{r} = \frac{x}{r^2}$

$$\Rightarrow \nabla\varphi = \frac{1}{r^2}(x,y,z) = \frac{1}{r^2}\underline{r}$$

In case (i), at $(-a,a,a)$,

$$\nabla\varphi = (-2a^5)\underline{i} + 0\underline{j} + (-2a^5)\underline{k} = -2a^5(\underline{i}+\underline{k})$$

From (1.2.5), $\nabla\varphi$ is normal to the surface $\varphi = c$, passing through $(-a,a,a)$. A unit vector normal to the surface is $\pm\frac{1}{\sqrt{2}}(\underline{i}+\underline{k})$.

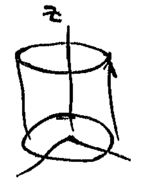
At $(-a,a,a)$, $\varphi = -a^6 + a^6 + 3a^6 = 3a^6 \Rightarrow c = 3a^6$.

3. $|\vec{OP}| = (x^2+y^2+z^2)^{\frac{1}{2}} = r$

(i) $x^2+y^2+z^2 = \varphi = c \Rightarrow |\vec{OP}| = r = c^{\frac{1}{2}}$, a constant.
$\Rightarrow P$ lies on a sphere of radius $c^{\frac{1}{2}}$.

(ii) Similarly, $(r^2)^{-\frac{3}{2}} = c \Rightarrow r = c^{-\frac{1}{3}}$, sphere, radius $c^{-\frac{1}{3}}$.

(iii) $x^2+y^2=c$

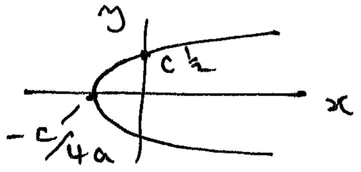


Infinite circular cylinder. Radius $c^{1/2}$, axis Oz.

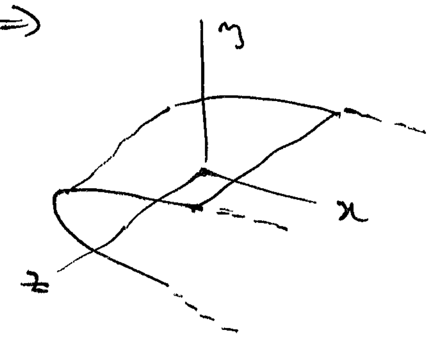
(iv) Circular cylinder, axis Ox, radius $c^{1/2}$.

(v) $y^2-4ax=c \Rightarrow y^2=4a\left(x+\frac{c}{4a}\right)$

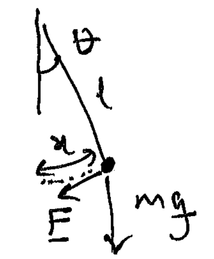
In the (x,y)-plane:



Independent of $z \Rightarrow$



4.



$|\underline{F}| = mg\sin\theta \approx mg\theta, \quad x=l\theta$

$= mgx/l.$

Also, $|\underline{F}| = ma = m\ddot{x}$

$\Rightarrow m\ddot{x} = -mgx/l \Rightarrow \ddot{x}+\frac{g}{l}x=0.$

restoring force

$\Rightarrow x = A\cos\omega t + B\sin\omega t$ with $\omega=\sqrt{g/l}$.

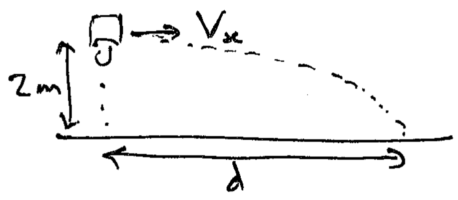
"Simple harmonic motion".

5.

At top, radial acc. must match gravity

$\Rightarrow \omega^2 r = g \Rightarrow \omega = \sqrt{g/r} \approx \sqrt{9.81}\ s^{-1}$

$= 3.13$ Hz



$V_x = \omega r = 3.13\ ms^{-1}$

Time to fall: $\frac{1}{2}gt^2 = 2m$

$\Rightarrow t = \sqrt{4/9.81}\,s \approx 0.639\,s$

Distance $d = v_x t = 3.13\ ms^{-1} \times 0.639\,s$

$\approx 2\,m$.