1. If $\mathbf{F} = (2x + 2y + 4z, 2x + 3y - 2z, 4x - 2y - 5z)$, find $\nabla \cdot \mathbf{F}$, $\nabla \times \mathbf{F}$, $\nabla^2\mathbf{F}$ and $(\mathbf{F} \cdot \nabla)\mathbf{F}$.
   Show that $\mathbf{F} = \nabla\phi$ for some ϕ to be determined.
   Verify that $\mathbf{F} = \nabla \times \mathbf{A}$, where
   $$\mathbf{A} = (y^2 - z^2 + 3yz + 2zx)\mathbf{i} + (2x^2 - 2z^2 - 2yz - 2zx)\mathbf{j}.$$

2. Find $\nabla^2(\ln r)$.

3. Given that $\mathbf{F} = (xy^2, yz^2, -x^3)$, find
   (i) $\nabla \cdot \mathbf{F}$; (ii) $\nabla(\nabla \cdot \mathbf{F})$; (iii) $\nabla \times \mathbf{F}$; (iv) $\nabla \times (\nabla \times \mathbf{F})$; (v) $\nabla^2\mathbf{F}$; (vi) $(\mathbf{F} \cdot \nabla)\mathbf{F}$.
   Verify that

   (1) $\nabla^2\mathbf{F} = \nabla(\nabla \cdot \mathbf{F}) - \nabla \times (\nabla \times \mathbf{F})$;
   (2) $(\mathbf{F} \cdot \nabla)\mathbf{F} = \nabla(\frac{1}{2}\mathbf{F}^2) - \mathbf{F} \times (\nabla \times \mathbf{F})$,

   where, in the usual convention, $\mathbf{F}^2 = \mathbf{F} \cdot \mathbf{F}$.

4. Given that $\phi = xf(r)$, calculate $\nabla^2\phi$. Verify that
   $$\nabla^2\phi = xr^{-4}\frac{d}{dr}\left(r^4\frac{df}{dr}\right).$$
   Hence determine the most general form for $f(r)$ that ensures that $\nabla^2\phi = 0$ for all $r \neq 0$.

5. If $b_{11} = 1,\ b_{12} = -1,\ b_{13} = 0,\ b_{21} = -2,\ b_{22} = 3,\ b_{23} = 1,\ b_{31} = 2,\ b_{32} = 0,\ b_{33} = 4,$ evaluate
   $$\text{(i) } b_{jj},\ \text{(ii) } b_{i1}b_{i2},\ \text{(iii) } b_{3p}b_{2p},\ \text{(vi) } \delta_{1i}b_{1i},\ \text{(v) } \delta_{jk}b_{2j}b_{3k},\ \text{(vi) } \epsilon_{1jk}b_{jk}.$$

6. Verify that
   $$\text{(i) } \delta_{ij}\delta_{ij} = 3; \qquad \text{(ii) } \delta_{ij}F_{ik} = F_{jk}.$$

7. Show that the i-component of $\nabla \times (\mathbf{u} \times \mathbf{v})$ is $\epsilon_{ijk}\frac{\partial}{\partial x_j}(\epsilon_{klm}u_l v_m)$. Show that this expression is equal to $\frac{\partial}{\partial x_j}(u_i v_j) - \frac{\partial}{\partial x_j}(u_j v_i)$.
   Hence prove the expansion formula $\nabla \times (\mathbf{u} \times \mathbf{v}) = (\mathbf{v} \cdot \nabla)\mathbf{u} - (\mathbf{u} \cdot \nabla)\mathbf{v} + \mathbf{u}(\nabla \cdot \mathbf{v}) - \mathbf{v}(\nabla \cdot \mathbf{u})$.

8. Using suffix notation, show that $\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$.