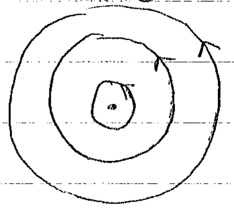
1 $\underline{u} = \Omega(-y, x, 0)$

$$\frac{\partial \psi}{\partial y} = -\Omega y \Rightarrow \psi = -\tfrac{1}{2}\Omega y^2 + f(x)$$

$$-\frac{\partial \psi}{\partial x} = \Omega x \Rightarrow \psi = -\tfrac{1}{2}\Omega x^2 + g(y)$$

TOGETHER, $\psi = -\tfrac{1}{2}\Omega(x^2+y^2) + \text{const}$.

STREAMLINES: $\psi = \text{const} \Rightarrow x^2+y^2 = \text{const}$



$$\underline{\omega} = \nabla \wedge \underline{u} = \Omega(0, 0, \partial_x x - \partial_y(-y))$$
$$= 2\Omega \underline{k}$$

VORTICITY CONSTANT, SAME EVERYWHERE

$\rightarrow$ SOLID BODY ROTATION.

2 $\underline{u} = U(-x, y, 0)$

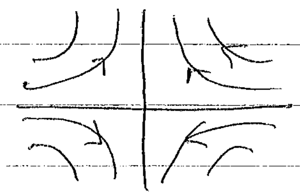
$$\frac{\partial \psi}{\partial y} = -Ux \Rightarrow \psi = -Uxy + f(x)$$

$$-\frac{\partial \psi}{\partial x} = Uy \Rightarrow \psi = -Uxy + g(y)$$

TOGETHER, $\psi = -Uxy + 0$ $\leftarrow$ $\psi = 0$ AT $x = y = 0$

STREAMLINES:

$\psi = \text{const} \Rightarrow y = \frac{A}{x}$ $A = \text{const}$.



(NOTE DIRECTION OF ARROWS)

$$\underline{\omega} = \nabla \wedge \underline{u} = U(0, 0, \partial_x y - \partial_y(-x)) = \underline{0}$$

$\Rightarrow$ IRROTATIONAL.

$\therefore \underline{u} = \nabla \phi$

$$\partial_x \phi = -Ux \Rightarrow \phi = -\tfrac{1}{2}Ux^2 + f(y)$$

$$\partial_y \phi = Uy \Rightarrow \phi = \tfrac{1}{2}Uy^2 + g(x)$$

TOGETHER, $\phi = \tfrac{1}{2}U(y^2 - x^2) + 0$

$\phi = 0$ AT $x = y = 0$.

BERNOULLI $\Rightarrow \frac{p}{\rho} = \underbrace{\underline{g}\cdot\underline{r}}_{\substack{0 \\ \text{NO BODY FORCE}}} - \underbrace{\partial_t \phi}_{\substack{0 \\ \text{INDEP. OF } t}} - \tfrac{1}{2}\underline{u}\cdot\underline{u} + \text{const}$ $\leftarrow \underline{u} = U(-x, y, 0)$

PRESSURE $p = -\tfrac{1}{2}\rho U^2(x^2+y^2) + \text{const}$.

$p$ LARGEST AT ORIGIN.

3 i) AS $r \to \infty$, $\frac{1}{r} \to 0 \Rightarrow \boxed{\underline{u} \to \underline{0}}$ AS $r \to \infty$.

ii) FOR $r=a$, $\underline{u} = \underline{W}(3+1) + (\underline{W}\cdot\underline{r})\underline{r}(3-3) = \boxed{4\underline{W}}$

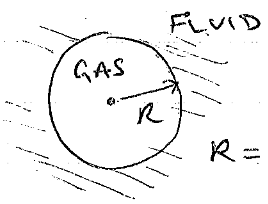
iii) $u_i = W_i\left(3\frac{a}{r} + \frac{a^3}{r^3}\right) + (W_j x_j)\, x_i \left(3\frac{a}{r^3} - 3\frac{a^3}{r^5}\right)$

$$\nabla\cdot\underline{u} = \partial_i u_i = W_i \frac{d}{dr}\left(3\frac{a}{r} + \frac{a^3}{r^3}\right)\frac{\partial r}{\partial x_i} + W_j\, \partial_i\left\{x_j x_i \left(3\frac{a}{r^3} - 3\frac{a^3}{r^5}\right)\right\}$$

$$= W_i\left(-3\frac{a}{r^2} - 3\frac{a^3}{r^4}\right)\frac{x_i}{r} + W_j\,\delta_{ij}\, x_i\left(3\frac{a}{r^3} - 3\frac{a^3}{r^5}\right) + W_j x_j\,\delta_{ii}\left(3\frac{a}{r^3} - 3\frac{a^3}{r^5}\right) + W_j x_j x_i\left(-9\frac{a}{r^4} + 15\frac{a^3}{r^6}\right)\frac{x_i}{r}$$

$$= W_i x_i\left(-3\frac{a}{r^3} - 3\frac{a^3}{r^5}\right) + W_i x_i\left(3\frac{a}{r^3} - 3\frac{a^3}{r^5}\right) + W_j x_j \cdot 3\left(3\frac{a}{r^3} - 3\frac{a^3}{r^5}\right) + W_j x_j\left(-9\frac{a}{r^4} + 15\frac{a^3}{r^5}\right)\frac{r^2}{r}$$

$$= W_i x_i \frac{a}{r^3}(-3+3+9-9) + W_i x_i \frac{a^3}{r^5}(-3-3-9+15) = \boxed{0}$$

4. 


$R = R(t)$

$W = -R^2\dot{R}/r$, $\underline{u} = \nabla W$

(SP.1) $\Rightarrow \underline{u} = -R^2\dot{R}\left[\partial_r\left(\frac{1}{r}\right)\right]\hat{\underline{r}} = \frac{R^2\dot{R}}{r^2}\hat{\underline{r}}$

BERNOULLI, NON-STEADY, $g \to \underline{0}$

$\Rightarrow \frac{p}{\rho} = -\partial_t W - \frac{1}{2}\underline{u}\cdot\underline{u} + \text{CONST}$,

$\partial_t W = -\frac{1}{r}\frac{d}{dt}(R^2\dot{R}) = -\frac{1}{r}(2R\dot{R}\dot{R} + R^2\ddot{R})$, $\underline{u}\cdot\underline{u} = \left(\frac{R^2\dot{R}}{r^2}\right)^2$

$r \to \infty \Rightarrow p_\infty/\rho = 0 + 0 + \text{CONST}$, I.E. $\text{CONST} = p_\infty/\rho$

$\therefore p = \frac{\rho}{r}(2R\dot{R}^2 + R^2\ddot{R}) - \frac{\rho}{2}\left(\frac{R^2\dot{R}}{r^2}\right)^2 + p_\infty$

ON $r = R$, $\boxed{p = p_\infty + \rho\left\{2\dot{R}^2 + R\ddot{R} - \frac{1}{2}\dot{R}^2\right\}}$

5 $\underline{u} = \nabla W$, $W = -\dfrac{Ua^3}{2r^2}\cos\theta$, SPs.

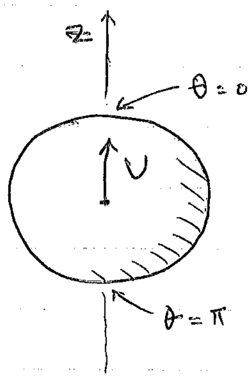
$$(SP.1) \Rightarrow \underline{u} = (\partial_r W)\,\hat{\underline{r}} + \left(\frac{1}{r}\partial_\theta W\right)\hat{\underline{\theta}}$$

$$\Rightarrow \underline{u} = \frac{Ua^3}{r^3}\cos\theta\,\hat{\underline{r}} + \frac{Ua^3}{2r^3}\sin\theta\,\hat{\underline{\theta}}$$

$r \longrightarrow \infty \Rightarrow \underline{u} \longrightarrow \underline{0}$. ON $r = a$ (SPHERE),

$$\underline{u}\cdot\hat{\underline{n}} = \underline{u}\cdot\hat{\underline{r}} = U\cos\theta$$

$\neq 0 \Rightarrow$ MOVING BOUNDARY




SPHERE MOVING ALONG Z-AXIS (FOR INSTANT WHEN CENTRED ON ORIGIN)

[COMPARE WITH W. Ex. 9.2]

$$\underline{u} = \nabla\wedge\left(\frac{\psi}{r}\hat{\underline{\phi}}\right)$$

$$(SP.3) \Rightarrow \underline{u} = \left(\frac{1}{r\sin\theta}\partial_\theta \sin\theta\cdot\frac{\psi}{r}\right)\hat{\underline{r}} - \left(\frac{1}{r\sin\theta}\partial_r \sin\theta\cdot\psi\right)\hat{\underline{\theta}}$$

$$= \left(\frac{1}{r^2\sin\theta}\partial_\theta\psi\sin\theta\right)\hat{\underline{r}} - \left(\frac{1}{r}\partial_r\psi\right)\hat{\underline{\theta}}$$

$\theta$ COMPONENTS:

$$-\frac{1}{r}\partial_r\psi = \frac{Ua^3}{2r^3}\sin\theta$$

$$\Rightarrow \partial_r\psi = -\frac{Ua^3}{2r^2}\sin\theta$$

$$\Rightarrow \psi = \frac{Ua^3}{2r}\sin\theta + f(\theta) \quad (1)$$

$r$ COMPONENTS:

$$\frac{1}{r^2\sin\theta}\partial_\theta\psi\sin\theta = \frac{Ua^3}{r^3}\cos\theta$$

$$\Rightarrow \partial_\theta\psi\sin\theta = \frac{Ua^3}{r}\sin\theta\cos\theta$$

$$\Rightarrow \psi\sin\theta = \frac{Ua^3}{2r}\sin^2\theta + g(r) \quad (2)$$

(1) AND (2) SAME WITH $f(\theta) = g(r) = 0$

$$\boxed{\psi = \frac{Ua^3\sin\theta}{2r}}$$