

X 1. Estimate Reynolds number  $R$  in the following cases:

- a submerged submarine moving at 15 knots;
- a man walking;
- a goldfish swimming in a large tank;
- a teaspoon sinking in syrup ( $\nu \approx 10^{-1} \text{ m}^2 \text{ s}^{-1}$ ,  $U \approx 5 \text{ mms}^{-1}$ ).

2. (a) Show that the equation of continuity  $\nabla \cdot \underline{u} = 0$  is satisfied identically by  $\underline{u} = \nabla \times [\psi(x, y) \hat{z}]$ . Express the vorticity equation in terms of  $\psi$ . Show also that  $(\underline{u} \cdot \nabla) \psi = 0$  and interpret this result.

(b) Repeat the steps in (a) in another case where  $\underline{u} = \nabla \times \left[ \frac{\psi(r, \theta)}{r \sin \theta} \hat{\phi} \right]$  with  $(r, \theta, \phi)$  being spherical polar coordinates.

X 3. In an infinite viscous fluid, the velocity field is steady and has components  $(\alpha x, -\alpha y, w(y))$ , where  $\alpha$  is a positive constant. Show that  $\underline{\omega} = (J, 0, 0)$ , where

$$\nu \frac{d^2 J}{dy^2} + \frac{d}{dy} (\alpha y J) = 0.$$

Find  $J$  given that  $J \rightarrow 0$  as  $|y| \rightarrow \infty$ , and interpret your result physically. Also find  $w(y)$  if  $w(y) \rightarrow \pm W$  as  $y \rightarrow \pm \infty$ .

4. It is observed that when the flow due to holding a circular cylinder of radius  $a$  at rest in a stream of uniform constant speed  $U$  at infinity (where the direction of the stream is perpendicular to the cylinder axis) has a Reynolds number lying in a certain range, regular oscillations occur in the flow behind the cylinder (the phenomenon being known as the von Kármán vortex street). How does the frequency of these oscillations change when  $a$  and  $U$ , but not the product  $aU$ , change? [HINT: This is a very short question using dimensional arguments only.]

X 5. Consider the steady flow of an inviscid fluid where there is a body force  $\underline{b} = -\nabla V$ . Show that

$$\nabla \left( \frac{1}{2} \underline{u}^2 \right) - \underline{u} \times \underline{\omega} = -\frac{1}{\rho} \nabla p - \nabla V.$$

If, further,  $p$  is uniform, deduce that  $\underline{u} \cdot \nabla H = \underline{\omega} \cdot \nabla H = 0$ , where  $H = \frac{p}{\rho} + V + \frac{1}{2} \underline{u}^2$ . What can be deduced from these results?

