

X 1. Estimate Reynolds number R in the following cases:

- (i) a submerged submarine moving at 15 knots;
- (ii) a man walking;
- (iii) a goldfish swimming in a large tank;
- (iv) a teaspoon sinking in syrup ($\nu \approx 10^{-1} \text{ m}^2 \text{ s}^{-1}$, $U \approx 5 \text{ mms}^{-1}$).

2. (a) Show that the equation of continuity $\nabla \cdot \underline{u} = 0$ is satisfied identically by $\underline{u} = \nabla \times [\psi(x, y) \hat{z}]$. Express the vorticity equation in terms of ψ . Show also that $(\underline{u} \cdot \nabla) \psi = 0$ and interpret this result.

(b) Repeat the steps in (a) in another case where $\underline{u} = \nabla \times \left[\frac{\psi(r, \theta)}{r \sin \theta} \hat{\phi} \right]$ with (r, θ, ϕ) being spherical polar coordinates.

X 3. In an infinite viscous fluid, the velocity field is steady and has components $(\alpha x, -\alpha y, w(y))$, where α is a positive constant. Show that $\underline{\omega} = (J, 0, 0)$, where

$$\nu \frac{d^2 J}{dy^2} + \frac{d}{dy} (\alpha y J) = 0.$$

Find J given that $J \rightarrow 0$ as $|y| \rightarrow \infty$, and interpret your result physically. Also find $w(y)$ if $w(y) \rightarrow \pm W$ as $y \rightarrow \pm \infty$.

4. It is observed that when the flow due to holding a circular cylinder of radius a at rest in a stream of uniform constant speed U at infinity (where the direction of the stream is perpendicular to the cylinder axis) has a Reynolds number lying in a certain range, regular oscillations occur in the flow behind the cylinder (the phenomenon being known as the von Kármán vortex street). How does the frequency of these oscillations change when a and U , but not the product aU , change? [HINT: This is a very short question using dimensional arguments only.]

X 5. Consider the steady flow of an inviscid fluid where there is a body force $\underline{b} = -\nabla V$. Show that

$$\nabla \left(\frac{1}{2} \underline{u}^2 \right) - \underline{u} \times \underline{\omega} = -\frac{1}{\rho} \nabla p - \nabla V.$$

If, further, p is uniform, deduce that $\underline{u} \cdot \nabla H = \underline{\omega} \cdot \nabla H = 0$, where $H = \frac{p}{\rho} + V + \frac{1}{2} \underline{u}^2$. What can be deduced from these results?

