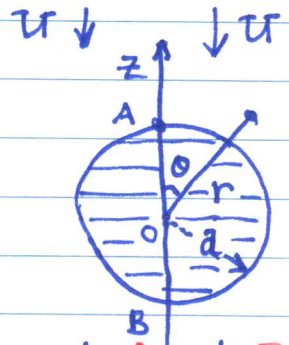


1. An infinite circular cylinder of radius  $a$  is held in moving fluid whose velocity far away is  $-U\hat{z}$  (see sketch). Show that the appropriate irrotational flow has velocity potential  $\phi = -Urcos\theta(1+a^2r^{-2})$ , where  $\underline{u} = \nabla\phi$ , and  $(r, \theta)$  are plane polar coordinates.



Show that the stagnation points (points where  $\underline{u} = \underline{0}$ ) are at  $A$  and  $B$ .

In the neighbourhood of  $A$ , write  $x = a\theta$ ,  $y = z - a$ . Draw a sketch to illustrate the geometrical meaning of  $x$  and  $y$  when  $x \ll a$ ,  $y \ll a$ , and show that then  $\phi \approx \phi_0 + \frac{1}{2}\alpha(x^2 - y^2)$ , where  $\phi_0$  and  $\alpha$  are constants.

Perform a similar analysis in the neighbourhood of  $B$ .

2. Consider the boundary layer on a flat plate in a uniform stream of speed  $U$  studied in §4.3. The stream function  $\psi$  is equal to  $U\ell f(\eta)$ , where  $\ell = \ell(x) = (2\nu x/U)^{1/2}$  and  $\eta = y/\ell$ ;  $f(\eta)$  satisfies  $f''' + ff'' = 0$  with  $f(0) = f'(0) = 0$ ,  $f'(\eta) \rightarrow 1$  as  $\eta \rightarrow \infty$ .

(i) Show that the first two non-zero terms in the Taylor series of  $f(\eta)$  about  $\eta = 0$  are  $\frac{1}{2}\beta\eta^2 - \frac{1}{120}\beta^2\eta^5$ , where  $\beta = f''(0)$ .

(ii) Prove that  $\frac{d^2}{d\eta^2}(\ln f'') = -f'$ ,

and deduce that there are constants  $k$  and  $\ell$  so that, for  $\eta \gg 1$ ,

$$f'(\eta) \approx 1 - \ell \int_{\eta}^{\infty} e^{-\frac{1}{2}(\xi-k)^2} d\xi.$$

(iii) Suppose  $g(\xi)$  satisfies  $g''' + gg'' = 0$  with  $g(0) = g'(0) = 0$ ,  $g''(0) = 1$ , and that  $g'(\xi) \rightarrow C^{-2}$  as  $\xi \rightarrow \infty$ . Show that  $f(\eta) = Cg(C\eta)$ .

3. Show that  $U\delta_1 = \int_0^{\infty} (U - u) dy$  is the reduction in the volumetric flow per unit width due to the boundary layer. Use this result with appropriate diagrams to explain why  $\delta_1 = \delta_1(x)$  is the distance through which streamlines of the external flow are displaced outwards by the boundary layer.

4. Verify that when  $U(x) = \alpha x$  and  $\ell = (\nu/\alpha)^{1/2}$ , where the constant  $\alpha$  is positive, (4.18) becomes  $f''' + ff'' - (f')^2 + 1 = 0$ , with  $f(0) = f'(0) = 0$  and  $f'(\eta) \rightarrow 1$  as  $\eta \rightarrow \infty$ .

Show that this solution of the boundary layer equations is also an exact

