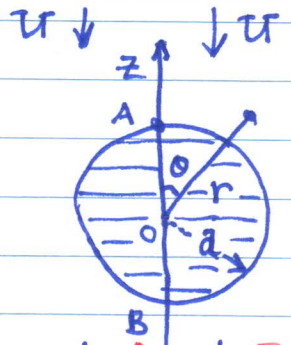


1. An infinite circular cylinder of radius a is held in moving fluid whose velocity far away is $-U\hat{z}$ (see sketch). Show that the appropriate irrotational flow has velocity potential $\phi = -Urcos\theta(1+a^2r^{-2})$, where $\underline{u} = \nabla\phi$, and (r, θ) are plane polar coordinates.



Show that the stagnation points (points where $\underline{u} = \underline{0}$) are at A and B .

In the neighbourhood of A , write $x = a\theta$, $y = z - a$. Draw a sketch to illustrate the geometrical meaning of x and y when $x \ll a$, $y \ll a$, and show that then $\phi \approx \phi_0 + \frac{1}{2}\alpha(x^2 - y^2)$, where ϕ_0 and α are constants.

Perform a similar analysis in the neighbourhood of B .

2. Consider the boundary layer on a flat plate in a uniform stream of speed U studied in §4.3. The stream function ψ is equal to $U\ell f(\eta)$, where $\ell = \ell(x) = (2\nu x/U)^{1/2}$ and $\eta = y/\ell$; $f(\eta)$ satisfies $f''' + ff'' = 0$ with $f(0) = f'(0) = 0$, $f'(\eta) \rightarrow 1$ as $\eta \rightarrow \infty$.

(i) Show that the first two non-zero terms in the Taylor series of $f(\eta)$ about $\eta = 0$ are $\frac{1}{2}\beta\eta^2 - \frac{1}{120}\beta^2\eta^5$, where $\beta = f''(0)$.

(ii) Prove that $\frac{d^2}{d\eta^2}(\ln f'') = -f'$,

and deduce that there are constants k and ℓ so that, for $\eta \gg 1$,

$$f'(\eta) \approx 1 - \ell \int_{\eta}^{\infty} e^{-\frac{1}{2}(\xi-k)^2} d\xi.$$

(iii) Suppose $g(\zeta)$ satisfies $g''' + gg'' = 0$ with $g(0) = g'(0) = 0$, $g''(0) = 1$, and that $g'(\zeta) \rightarrow C^{-2}$ as $\zeta \rightarrow \infty$. Show that $f(\eta) = Cg(C\eta)$.

3. Show that $U\delta_1 = \int_0^{\infty} (U - u) dy$ is the reduction in the volumetric flow per unit width due to the boundary layer. Use this result with appropriate diagrams to explain why $\delta_1 = \delta_1(x)$ is the distance through which streamlines of the external flow are displaced outwards by the boundary layer.

4. Verify that when $U(x) = \alpha x$ and $\ell = (\nu/\alpha)^{1/2}$, where the constant α is positive, (4.18) becomes $f''' + ff'' - (f')^2 + 1 = 0$, with $f(0) = f'(0) = 0$ and $f'(\eta) \rightarrow 1$ as $\eta \rightarrow \infty$.

Show that this solution of the boundary layer equations is also an exact

