OPERATORS IN CYLINDRICAL POLARS (CPs) AND SPHERICAL POLARS (SPs)

1. CPs (Cylindrical Polars) $\quad (r, \theta, z) \quad h_1 = 1,\ h_2 = r,\ h_3 = 1$

$$\nabla V = \frac{\partial V}{\partial r}\ \hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\ \hat{\boldsymbol{\theta}} + \frac{\partial V}{\partial z}\ \hat{\boldsymbol{z}} \tag{CP.1}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r}\ \frac{\partial}{\partial r}(rF_1) + \frac{1}{r}\frac{\partial F_2}{\partial \theta} + \frac{\partial F_3}{\partial z} \tag{CP.2}$$

$$\nabla \times \mathbf{F} = \left[\frac{1}{r}\frac{\partial F_3}{\partial \theta} - \frac{\partial F_2}{\partial z}\right]\hat{\boldsymbol{r}} + \left[\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial r}\right]\hat{\boldsymbol{\theta}} + \left[\frac{1}{r}\frac{\partial}{\partial r}(rF_2) - \frac{1}{r}\frac{\partial F_1}{\partial \theta}\right]\hat{\boldsymbol{z}} \tag{CP.3}$$

$$\nabla^2 V = \frac{1}{r}\ \frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\ \frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2} \tag{CP.4}$$

2. SPs (Spherical Polars) $\quad (r, \theta, \phi) \quad h_1 = 1,\ h_2 = r,\ h_3 = r\sin\theta$

$$\nabla V = \frac{\partial V}{\partial r}\ \hat{\boldsymbol{r}} + \frac{1}{r}\ \frac{\partial V}{\partial \theta}\ \hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\ \frac{\partial V}{\partial \phi}\ \hat{\boldsymbol{\phi}} \tag{SP.1}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r^2}\ \frac{\partial}{\partial r}(r^2 F_1) + \frac{1}{r\sin\theta}\ \frac{\partial}{\partial \theta}\ \{(\sin\theta)F_2\} + \frac{1}{r\sin\theta}\ \frac{\partial F_3}{\partial \phi} \tag{SP.2}$$

$$\nabla \times \mathbf{F} = \frac{1}{r\sin\theta}\ \left[\frac{\partial}{\partial \theta}\{(\sin\theta)F_3\} - \frac{\partial F_2}{\partial \phi}\right]\hat{\boldsymbol{r}} + \frac{1}{r\sin\theta}\left[\frac{\partial F_1}{\partial \phi} - \frac{\partial}{\partial r}\{(r\sin\theta)F_3\}\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_2) - \frac{\partial F_1}{\partial \theta}\right]\hat{\boldsymbol{\phi}} \tag{SP.3}$$

$$\nabla^2 V = \frac{1}{r^2}\ \frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\ \frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} \tag{SP.4}$$