VECTOR CALCULUS IDENTITIES

$$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi \tag{E.1}$$

$$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B} \tag{E.2}$$

$$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B} \tag{E.3}$$

$$\nabla(\phi\,\psi) = \phi\,\nabla\psi + \psi\,\nabla\phi \tag{E.4}$$

$$\nabla \cdot (\phi\,\mathbf{v}) = \phi\,\nabla \cdot \mathbf{v} + (\mathbf{v} \cdot \nabla)\phi \tag{E.5}$$

$$\nabla \times (\phi\,\mathbf{v}) = \phi\,\nabla \times \mathbf{v} + \nabla\phi \times \mathbf{v} \tag{E.6}$$

$$\nabla \cdot (\mathbf{u} \times \mathbf{v}) = \mathbf{v} \cdot \nabla \times \mathbf{u} - \mathbf{u} \cdot \nabla \times \mathbf{v} \tag{E.7}$$

$$\nabla \times (\mathbf{u} \times \mathbf{v}) = (\mathbf{v} \cdot \nabla)\mathbf{u} - (\mathbf{u} \cdot \nabla)\mathbf{v} + \mathbf{u}(\nabla \cdot \mathbf{v}) - \mathbf{v}(\nabla \cdot \mathbf{u}) \tag{E.8}$$

$$\nabla(\mathbf{u} \cdot \mathbf{v}) = (\mathbf{v} \cdot \nabla)\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{v} + \mathbf{v} \times (\nabla \times \mathbf{u}) + \mathbf{u} \times (\nabla \times \mathbf{v}) \tag{E.9}$$

- (E.1-E.3) express the linearity property of the vector operators.
- (E.4-E.7) follow immediately using subscript notation and the product rule. You should know or be able to quickly derive them, e.g.

$$\nabla \cdot (\phi\mathbf{v}) = \partial_i(\phi v_i) = \phi\,\partial_i v_i + v_i \partial_i \phi = \phi\,\nabla \cdot \mathbf{v} + (\mathbf{v} \cdot \nabla)\,\phi\,.$$

- (E.8-E.9) you should be able to derive, given the identity

$$\epsilon_{ijk}\,\epsilon_{klm} = \delta_{il}\,\delta_{jm} - \delta_{im}\,\delta_{jl}\ .$$